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A New Shape Function for Analysis of Line Continuum By Direct Variational Calculus

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Abstract

Trigonometric and polynomial functions are generally used in classical analysis of line continuum. Trigonometric functions tend to be good for direct use in the governing differential equations as well as in direct variational calculus. On the other hand, polynomial functions are only good for use in direct variational calculus because they give trivial results when used directly in governing differential equations. This explains the general resort to trigonometric functions in classical flexural linear continuum analysis. However, the problem with trigonometric functions is that it is very difficult to satisfy the boundary conditions of a propped cantilever linear continuum when a trigonometric shape function is used. Thus, many trial functions based on trigonometric functions, polynomial functions, and combinations of both are currently in use, none of which is effective for all cases of line continuum analysis. This work presents a new polynomial shape function derived from the exact general solutions of the governing differential equations that would be suitable for use in direct variational calculus for analyzing all the basic forms of line continuum. A general polynomial shape function for linear continuum is first developed from the basic governing differential equations. Peculiar polynomial shape functions were then developed for four different cases of linear continua, namely pin–roller supports, clamp–roller supports, clamp–clamp supports, and clamp–free supports by satisfying their boundary conditions in the general shape function. These peculiar polynomial shape functions were applied in analyzing pure bending, free vibration, and buckling of line continuum by direct variational calculus. The results were found to be identical or very close to the exact results obtained using standard trigonometric shape functions in equilibrium approach, the percentage differences being 0% for pure bending analysis, 0-6.383% for buckling analysis, and 0-2.52% for free vibration analysis. These results confirm that the new polynomial shape function developed in this work is effective for analysis of all cases of line continuum with different boundary conditions. It is therefore recommended for use by structural analysts.

***Key words**: Shape function, Variational calculus, Line continuum, Differential equation, Polynomial, Trigonometric*

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I. INTRODUCTION

Trigonometric and polynomial functions have their merits and demerits when used in classical analysis of line continuum. Trigonometric functions give two results, one trivial and the other non-trivial, when used directly in the governing differential equations or in minimized total potential energy functionals, which is a form of direct variational calculus. On the other hand, polynomial functions also give non-trivial results when used in minimized total potential energy functionals but are weak when used directly in governing differential equations as they give trivial results. This weakness in polynomial functions when used directly in governing differential equations may be the reason for the general resort to trigonometric functions in classical flexural linear continuum analysis.However, although trigonometric functions are good for use in differential equations, they still have the problem that it is very difficult to satisfy the boundary conditions of a propped cantilever linear continuum (C – R line continuum) when a trigonometric shape function is used; attempting to do so results in a trivial peculiar solution (that is w = 0). Therefore, even analysts who are used to trigonometric functions still revert to some form of polynomial shape functions when a propped cantilever linear continuum is involved (El Naschie, 1990; Iyengar, 1988; Kassimali, 2011; Ghali et al., 2009). Thus, many trial functions based on trigonometric functions, polynomial functions, and combinations of both are currently in use, none of which is effective for all cases of line continuum analysis. In fact, the search for suitable shape functions has dominated classical and numerical research in solid mechanics, especially in recent times. This work aims at providing a new polynomial shape function derived from the exact general solutions of the governing differential equations that would be suitable for use in direct variational calculus for analyzing all the basic forms of line continuum.

II. BASIC EQUATIONS

The characteristic of a bent line continuum is defined with a fourth order differential equation given byUgural (1999) for a beam in pure bending as in equation (1).

$$EI\frac{d^4w}{dx^2} = P \qquad (1)$$

The governing equation for a slender columnin buckling is also of fourth order, given by Iyengar (1988) as in equation (2).

$$EI\frac{d^4w}{dx^4} + N\frac{d^2w}{dx^2} = 0 \qquad (2)$$

Chakraverty (2009) has also givena fourth order governing differential equation for a free vibrating beam as in equation (3).

$$EI\frac{d^4w}{dx^4} - \rho A\lambda^2 w = 0 \qquad (3)$$

Where EI, ρ, and A are flexural stiffness, density, and cross-section area of the member respectively;w and λ are the deflection and natural frequency of vibration respectively;Pis the uniform distributed load taken as constant load per meter length,and N is the axial load.

Equations (1), (2), and (3)should all have unique solutions because they are all unique. Since the load of equation (1) is uniformly distributed, the solution can readily be taken as in equation (4).

$$w = c_0 + \frac{c_1x}{EI} + \frac{c_2x^2}{2EI} + \frac{c_3x^3}{6EI} + \frac{Px^4}{24EI} \qquad (4)$$

Considering cases where the load may not always be uniform, the solution of equation (1) will generally be as stated in equation (5).

$$w = c_0 + \frac{c_1x}{EI} + \frac{c_2x^2}{2EI} + \frac{c_3x^3}{6EI} + \frac{1}{EI}\int\int\int\int Pdxdxdxdx \qquad (5)$$

It can be seen from equations (4) and (5) that the solution of equation (1) is in the form of a finite power series that could be truncated at about the fifth term.

Equations (2) and (3) cannot be easily integrated directly as equation (1). Assume an exponential solution of the form shown in equation (6a).

$$w = a_1 + a_2x + e^{hx}, \text{or } w = e^{hx} \qquad (6a)$$

Let $N = EIk^2$ in equation (2); i. e. $k^2 = \frac{N}{EI}$; then the general solution of equation (2) is as shown in equation (6b) which can also be written as in equation (6c).

$$w = a_1 + a_2x + a_3e^{ikx} + a_4e^{-ikx} \qquad (6b)$$

$$w = a_3e^{ikx} + a_4e^{-ikx} \qquad (6c)$$

where ikx is a complex number.

Similarly, let $w = e^{hx}$ be the solution of equation (3).

Substituting $k^4 = \rho A\lambda^2$ in the equation,the general solution of the differential equation is as shown in equation (7) (Goodwine, 2010; Bird, 2010; James et al, 2011).

$$w = a_1e^{kx} + a_2e^{-kx} + a_3e^{ikx} + a_4e^{-ikx} \qquad (7)$$

Equations (6b), (6c), and (7) are exponential functions. In accordance with Stroud (1982), Goodwine (2010), Bird (2010), and James et al. (2011), these equations can be transformed into trigonometric functions as shown in equations (8) to (11) or into polynomial functions (power series) as shown in equations (12) to (15).

$$e^{ikx} = \text{Cos } kx + i\text{Sin } kx \qquad (8)$$

$$e^{-ikx} = \text{Cos } kx - i\text{Sin } kx \qquad (9)$$

$$e^{kx} = \text{Cosh } kx + \text{Sinh } kx \qquad (10)$$

$$e^{-kx} = \text{Cosh } kx - \text{Sinh } kx \qquad (11)$$

$$e^{kx} = 1 + kx + \frac{(kx)^2}{2!} + \frac{(kx)^3}{3!} + \frac{(kx)^4}{4!} + \dots \qquad (12)$$

$$e^{-kx} = 1 - kx + \frac{(kx)^2}{2!} - \frac{(kx)^3}{3!} + \frac{(kx)^4}{4!} - \dots \qquad (13)$$

$$e^{ikx} = \left(1 - \frac{(kx)^2}{2!} + \frac{(kx)^4}{4!} - \dots\right) + i\left(kx - \frac{(kx)^3}{3!} + \frac{(kx)^5}{5!} - \dots\right) \qquad (14)$$

$$e^{-ikx} = \left(1 + \frac{(kx)^2}{2!} - \frac{(kx)^4}{4!} + \ldots\right) + i\left(-kx + \frac{(kx)^3}{3!} - \frac{(kx)^5}{5!} + \ldots\right) \quad (15)$$

Thus, the general solutions of equations (2) and (3)can be transformed from exponential forms to either trigonometric or polynomial forms. Applyingthe trigonometric equations (8), (9), (10), and (11) as required, the exponentialequations (6b), (6c), and (7)can be transformed into trigonometric general shape functions as shown in equations (16a), (16b), and (17).

$$w = c_1 + c_2 x + c_3 \mathrm{Cos}\, kx + c_4 \mathrm{Sin}\, kx \quad (16a)$$

$$w = c_1 \mathrm{Cos}\, kx + c_2 \mathrm{Sin}\, kx \quad (16b)$$

$$w = c_1 \mathrm{Cosh}\, kx + c_1 \mathrm{Sinh}\, kx + c_3 \mathrm{Cos}\, kx + c_4 \mathrm{Sin}\, kx \quad (17)$$

2.1 GENERAL POLYNOMIAL SHAPE FUNCTIONFOR LINEAR CONTINUUM

Substitution of equations (14) and (15) into equation (6b) results in equation (18):

$$w = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + \ldots \quad (18)$$

where $c_0 = a_1 + a_3 + a_4$; $c_1 = a_2 + ia_3 k - ia_4 k$; $c_2 = -a_3 \frac{k^2}{2!} + a_4 \frac{k^2}{2!}$;
$c_3 = -ia_3 \frac{k^3}{3!} + ia_4 \frac{k^3}{3!}$; $c_4 = a_3 \frac{k^4}{4!} - a_4 \frac{k^4}{4!}$; $c_5 = -ia_3 \frac{k^5}{5!} + ia_4 \frac{k^5}{5!}$; etc

Similarly, substituting equations (14) and (15) into equation (6c) results in equation (19):

$$w = d_0 + d_1 x + d_2 x^2 + d_3 x^3 + d_4 x^4 + d_5 x^5 + \ldots \quad (19)$$

where $d_0 = a_3 + a_4$; $d_1 = ia_3 - ia_4$; $d_2 = -a_3 \frac{k^2}{2!} + a_4 \frac{k^2}{2!}$; $d_3 = -ia_3 \frac{k^3}{3!} + ia_4 \frac{k^3}{3!}$;
$d_4 = a_3 \frac{k^4}{4!} - a_4 \frac{k^4}{4!}$; $d_5 = ia_3 \frac{k^5}{5!} - ia_4 \frac{k^5}{5!}$; etc

Substitution of equations (12), (13), (14) and (15) into equation (7) results in equation (20):

$$w = f_0 + f_1 x + f_2 x^2 + f_3 x^3 + f_4 x^4 + f_5 x^5 + \ldots \quad (20)$$

where $f_0 = a_1 + a_2 + a_3 + a_4$; $f_1 = k(a_1 + a_2 + ia_3 - ia_4)$; $f_2 = \frac{k^2}{2!}(a_1 + a_3 - a_3 + a_4)$;
$f_3 = \frac{k^3}{3!}(a_1 - a_2 - ia_3 + ia_4)$; $f_4 = \frac{k^4}{4!}(a_1 + a_2 + a_3 - a_4)$; $f_5 = \frac{k^5}{5!}(a_1 - a_2 + a_3 - a_4)$; etc

It can be seen that equations (5), (18), (19), and (20) are identical. These equations can be written as shown in equation (21):

$$w = \sum_{\alpha=0}^{\infty} b_\alpha x^\alpha \quad (21)$$

Equation (21) is the general polynomial shape function.It gives the general solution of a flexural line continuum in the form of infinite power series.

2.2 PECULIAR POLYNOMIALSHAPE FUNCTIONS FOR LINEAR CONTINUAWITH VARIOUS BOUNDARY CONDITIONS

Satisfying the boundary conditions of a particular line continuum in equation 21 will result in the exact peculiar shape function for the continuum in question. There are four boundary conditions for a flexural line continuum in solid mechanics, two at each end of the continuum. However, the number of terms in the general polynomial shape function is infinite. Hence, satisfying four conditions will also give a peculiar shape function with infinite number of terms. The nature of the general polynomial shape function of equation (21) as can be seen in equations (5), (18), (19), and (20) suggests that it converges at a finite or definite number of terms. If the infinite shape function is truncated at nth term and the four boundary conditions are satisfied, then the peculiar shape function will have (n – 4) degrees of freedom. If the function converges at nth term then it becomes a finite series as expressed in equation (22).

$$w = \sum_{\alpha=0}^{n} b_\alpha x^\alpha \quad (22)$$

It is always better to check the convergence of the seriesbeginning with one or two degrees of freedom corresponding to truncating the shape function at the fifth and sixth terms respectively, as shown in equations (23) and (24).

$$w = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4 \quad (23)$$

$$w = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4 + b_5 x^5 \quad (24)$$

Four linear continua with different boundary conditions were studied in this work, namelypin – roller supports (P–R), clamp – roller supports (C–R), clamp – clamp supports (C–C), and clamp – free supports (C–F). Their boundary conditions are shown in table 1. Satisfying these boundary conditions in equations (23) and (24) yield the peculiar shape functions presented in table 2.

Table 1: Four line continua and their boundary conditions

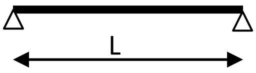
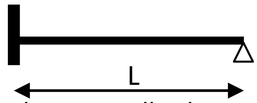
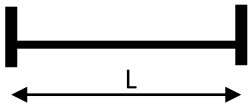
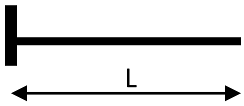
Line Continuum	Boundary Conditions	Line Continuum	Boundary Conditions
 Pin – Roller beam P-R	$W(0) = 0, \frac{d^2w(0)}{dx^2} = 0;$ $W(L) = 0, \frac{d^2w(L)}{dx^2} = 0$	 Clamp – Roller beam C-R	$W(0) = 0, \frac{dw(0)}{dx} = 0;$ $W(L) = 0, \frac{d^2w(L)}{dx^2} = 0$
 Clamp – Clamp beam C-C	$W(0) = 0, \frac{dw(0)}{dx} = 0;$ $W(L) = 0, \frac{dw(L)}{dx} = 0$	 Clamp – free beam C-F	$W(0) = 0, \frac{dw(0)}{dx} = 0;$ $M(L) = 0, V(L) = 0$
Legend: M =Bending moment; V = Shear force			

Table 2: Peculiar shape functions of flexural line continua of one and two degrees of freedom

Line Continuum	One degree of freedom	Two degrees of freedom
P – R line continuum	$w = b_4(R - 2R^3 + R^4)$	$w = b_4(R - 2R^3 + R^4) + b_5\left(\frac{7}{3}R - \frac{10}{3}R^3 + R^5\right)$
C – R line continuum	$w = b_4(1.5R^2 - 2.5R^3 + R^4)$	$w = b_4(1.5R^2 - 2.5R^3 + R^4) + b_5(3.5R^2 - 4.5R^3 + R^5)$
C – C line continuum	$w = b_4(R^2 - 2R^3 + R^4)$	$w = b_4(R^2 - 2R^3 + R^4) + b_5(2R^2 - 3R^3 + R^5)$
C – F line continuum For pure bending or free vibration analysis	$w = b_4(6R^2 - 4R^3 + R^4)$	$w = b_4(6R^2 - 4R^3 + R^4) + b_5(20R^2 - 10R^3 + R^5)$
C – F line continuum For instability buckling analysis	$w = b_4(8DR^2 - 6R^2 - \frac{8D}{3}R^3 + R^4)$ Where D = -1.1124	$w = b_4\left(8DR^2 - 6R^2 - \frac{8D}{3}R^3 + R^4\right) + b_5(15ER^2 - 10R^2 - 5ER^3 + R^5)$ Where D = -1.07200257 E = -3.14400514
Legend: $R = \frac{x}{L}$		

2.3 APPLICATION OF THE PECULIAR POLYNOMIAL SHAPE FUNCTIONS IN LINE CONTINUUM ANALYSIS BY DIRECT VARIATIONAL CALCULUS

The peculiar polynomial shape functions of one and two degrees of freedom in table 2 can be applied in analyzing pure bending, free vibration, and buckling of line continuum by direct variational calculus. The direct variational calculus functional to be used is the total potential energy fuctional given by El-Naschie (1990) as shown in equations (25), (26), and (27). These are the total potential energy functional for pure bending, buckling, and free vibration analyses of flexural line continuum respectively.

$$\pi = \frac{EI}{2}\int_0^L \frac{d^2w}{dx^2}dx - P\int_0^L w\,dx \qquad (25)$$

$$\pi = \frac{EI}{2}\int_0^L \frac{d^2w}{dx^2}dx - \frac{N}{2}\int_0^L \frac{dw}{dx}dx \qquad (26)$$

$$\pi = \frac{EI}{2}\int_0^L \frac{d^2w}{dx^2}dx - \frac{\rho A\omega^2}{2}\int_0^L w^2\,dx \qquad (27)$$

Using the non-dimensionalparameter, $R = \frac{X}{L}$, equations (25), (26), and (27) can be written as shown in equations(28), (29), and (30).

$$\pi = \frac{EI}{2L^3}\int_0^1 \left(\frac{d^2w}{dR^2}\right)^2 dR - PL\int_0^1 w\,dR \qquad (28)$$

$$\pi = \frac{EI}{2L^3}\int_0^1 \left(\frac{d^2w}{dR^2}\right)^2 dR - \frac{N}{2L}\int_0^L \left(\frac{dw}{dR}\right)^2 dR \qquad (29)$$

$$\pi = \frac{EI}{2L^3}\int_0^1 \left(\frac{d^2w}{dR^2}\right)^2 dR - \frac{\rho AL\omega^2}{2}\int_0^L w^2\,dR \qquad (30)$$

Substituting the peculiar polynomial shape functions of one and two degrees of freedom from the four linear continua of table 1 into equations (28), (29), and (30) and minimizing the resulting functions gives results for pure bending analysis, buckling analysis, and free vibration analysis as presented intables 3, 4, and 5 respectively. Exact results obtained using standard trigonometric shape functions in equilibrium approachare also shown in the tables for comparison with results from this workusing direct variational calculus.

Table 3: Results for pure bending analysis

Line Continuum	Centre deflection, $w\left(\frac{1}{2}\right)$		
	Exact Result	Result from this work	
		One degree of freedom	Two degrees of freedom
P – R	$\frac{5PL^4}{384EI}$	$w\left(\frac{1}{2}\right) = \frac{PL^4}{24EI}\left(\frac{5}{16}\right) = \frac{5PL^4}{384EI}$	$w\left(\frac{1}{2}\right) = \frac{PL^4}{24EI}\left(\frac{5}{16}\right) + 0\left(\frac{25}{32}\right) = \frac{5PL^4}{384EI}$
C – C	$\frac{PL^4}{384EI}$	$w\left(\frac{1}{2}\right) = \frac{PL^4}{24EI}\left(\frac{1}{16}\right) = \frac{PL^4}{384EI}$	$w\left(\frac{1}{2}\right) = \frac{PL^4}{24EI}\left(\frac{1}{16}\right) + 0\left(\frac{5}{32}\right) = \frac{PL^4}{384EI}$
C – R	$\frac{2PL^4}{384EI}$	$w\left(\frac{1}{2}\right) = \frac{PL^4}{24EI}\left(\frac{1}{8}\right) = \frac{2PL^4}{384EI}$	$w\left(\frac{1}{2}\right) = \frac{PL^4}{24EI}\left(\frac{1}{8}\right) + 0\left(\frac{11}{32}\right) = \frac{2PL^4}{384EI}$
C – F	$\frac{17PL^4}{384EI}$	$w\left(\frac{1}{2}\right) = \frac{PL^4}{24EI}\left(\frac{17}{16}\right) = \frac{17PL^4}{384EI}$	$w\left(\frac{1}{2}\right) = \frac{PL^4}{24EI}\left(\frac{17}{16}\right) + 0\left(\frac{121}{32}\right) = \frac{17PL^4}{384EI}$

Table 4: Results for buckling analysis

Line Continuum	Critical buckling load, P_{cr}		
	Exact Result	Result from this work	
		One degree of freedom	Two degrees of freedom
P – R	$P_{cr} = 9.8696\frac{EI}{L^2}$	$P_{cr} = 9.8824\frac{EI}{L^2}$	$P_{cr} = 9.8824\frac{EI}{L^2}$
C – C	$P_{cr} = 39.4784\frac{EI}{L^2}$	$P_{cr} = 41.9992\frac{EI}{L^2}$	$P_{cr} = 41.9991\frac{EI}{L^2}$
C – R	$P_{cr} = 20.191\frac{EI}{L^2}$	$P_{cr} = 21\frac{EI}{L^2}$	$P_{cr} = 20.3475\frac{EI}{L^2}$
C – F	$P_{cr} = 2.4674\frac{EI}{L^2}$	$P_{cr} = 2.4734\frac{EI}{L^2}$	$P_{cr} = 2.4826\frac{EI}{L^2}$

Table 5: Results for free vibration analysis

Line Continuum	Natural frequency of the continuum, ω		
	Exact Result	Result from this work	
		One degree of freedom	Two degrees of freedom
S – R	$\omega = \frac{9.8697}{L^2}\sqrt{\frac{EI}{\rho A}}$	$\omega = \frac{9.8767}{L^2}\sqrt{\frac{EI}{\rho A}}$	$\omega = \frac{10.1188}{L^2}\sqrt{\ }$
C - C	$\omega = \frac{22.3733}{L^2}\sqrt{\frac{EI}{\rho A}}$	$\omega = \frac{22.4521}{L^2}\sqrt{\frac{EI}{\rho A}}$	$\omega = \frac{22.5283}{L^2}\sqrt{\ }$
C – R	$\omega = \frac{15.4182}{L^2}\sqrt{\frac{EI}{\rho A}}$	$\omega = \frac{15.4511}{L^2}\sqrt{\frac{EI}{\rho A}}$	$\omega = \frac{15.44235}{L^2}$
C - F	$\omega = \frac{3.5156}{L^2}\sqrt{\frac{EI}{\rho A}}$	$\omega = \frac{3.5301}{L^2}\sqrt{\frac{EI}{\rho A}}$	$\omega = \frac{3.5671}{L^2}\sqrt{\frac{E}{\rho}}$

III. DISCUSSION OF RESULTS AND CONCLUSIONS

It can be seen from table 3 that the results for pure bending analysis from this work using direct variational calculus are the same as the exact results obtained using standard trigonometric shape functions in equilibrium approach. It is interesting to also note that theresults for two degrees of freedom shape function are the same as those for one degree. This should be expected since, for uniformly distributed load, direct integration of equation (1) gave equation (4), which is a truncation of the infinite series at the fifth term $(w = b_0 + b_1x + b_2x^2 + b_3x^3 + b_4x^4)$. Table 4also shows that the results for buckling analysis from this work are virtually the same as the exact results, the highest percentage difference being only 6.383% for C – C continuum. The results of this work for both one degree and two degrees of freedom are upper bound to the exact results, this being one of the characteristics of direct variational principle. These results suggest that the infinite series could be truncated at the fifth or sixth term to get the exact finite polynomial shape function for linear continuum buckling analysis.Table 5 further shows that the results for free vibration analysisfrom this work are similar to the exact results, the highest percentage difference being only2.52% for two degree of freedom shape function of P – R continuum. This confirms that the infinite series actually converges at the fifth term.Therefore, it can be safely concluded that truncating the infinite polynomial series at the fifth term gives results that are identical or very close to the exact results. Thus, the fifth term finite polynomial series can be taken as the exact shape function when transforming the exponential shape function into polynomial shape function. Finally, the results confirm that the new polynomial shape function developed in this work is effective for analysis of all cases of line continuum with different boundary conditions. It is therefore recommended for use by structural analysts.

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